

On the Coupling Constant for $N^*(1535)N\rho$

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The value of the $N^*(1535)N\rho$ coupling constant $g_{N^*N\rho}$ derived from the $N^*(1535) \rightarrow N\rho \rightarrow N\pi\pi$ decay is compared with that deduced from the radiative decay $N^*(1535) \rightarrow N\gamma$ using the vector-meson-dominance model. On the basis of an effective Lagrangian approach, we show that the values of $g_{N^*N\rho}$ extracted from the available experimental data on the two decays are consistent, though the error bars are rather large.

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I. INTRODUCTION

The experimental database on the production of the η meson in nucleon-nucleon scattering near threshold has expanded significantly in recent years. In addition to measurements of $pp \rightarrow pp\eta$ total cross sections and angular distributions [1], there are analyzing powers [2] and full Dalitz plots [3]. Total cross sections are also available for the $pn \rightarrow d\eta$ and $pn \rightarrow pn\eta$ reactions [4].

In response to this wealth of data there have been a large number of theoretical investigations of η production in both proton-proton and proton-neutron collisions. Most of these have been within the framework of meson-exchange models, where a $N^*(1535)$ resonance or other nucleon isobar is excited through the exchange of a single meson, with the η -meson being formed through the decay of the isobar. There are differences in the literature on how to treat the initial and final state interactions but the major controversies are connected with which meson exchanges are deemed to be important.

The large ratio of the production of the η in proton-neutron compared to proton-proton collisions suggests that isovector exchange plays the major role. However, some authors [5, 6, 7] find pseudoscalar (π and η) exchanges to dominate, with no significant contribution from the ρ . In contrast, others [8, 9, 10, 11, 12] claim that ρ -meson exchange plays an important and possibly dominant role. This disagreement is generated principally by the uncertainty in the size of the $N^*(1535)N\rho$ coupling and it is the purpose of this present note to compare the values of the coupling constant derived from the $N^*(1535) \rightarrow N\pi\pi$ and $N^* \rightarrow N\gamma$ decays.

The situation is further complicated by the variety of forms chosen for the $N^*(1535)N\rho$ coupling in these different works. In the vector meson dominance model (VMD), it is assumed that this coupling is proportional to that for the electromagnetic $N^*(1535)N\gamma$. It should

be noted that in this approach the tensor $\sigma_{\mu\nu}$ coupling automatically satisfies the associated gauge invariance constraint [6]. In contrast, the vector $\gamma_5\gamma_\mu$ coupling violates gauge invariance when the ρ -meson is replaced by a photon [6, 11]. As an alternative, Riska and Brown [13] suggested a vertex of the form $\gamma_5[\gamma^\mu p_\rho^2 - (M_{N^*} + m_N)p_\rho^\mu]$, where p_ρ is the four-momentum of the ρ meson. This avoids the gauge invariance problem while keeping the γ_μ term, but this coupling vanishes when used in connection with the VMD approach. In principle both vector and tensor couplings are needed and their relative importance has to be decided by experiment.

Working within an effective Lagrangian approach, we have investigated the influence of the $N^*(1535)N\rho$ coupling constant on both the $N^*(1535) \rightarrow N\rho^0 \rightarrow N\pi^+\pi^-$ and the $N^*(1535) \rightarrow N\rho^0 \rightarrow N\gamma$ decays. In Sect. II, we present the formalism and ingredients necessary for our estimations. Although in one case the ρ -meson is essentially real while in the other it has zero mass, we show in Sect. III that consistent values of the coupling constant can be obtained from the available experimental data on the two decay channels, though the uncertainties are still quite large.

II. FORMALISM AND APPLICATION

The basic Feynman diagrams for the two cascade decay modes considered here are depicted in Fig. 1. A Lorentz covariant orbital-spin (L - S) scheme for N^*NM couplings has been developed in detail in Ref. [14] and, within that scheme, one can easily derive the form of the effective $N^*(1535)N\rho$ coupling. Since the ρ is a vector meson, both S - and D -wave couplings are possible but experiment shows that the D -wave plays only an insignificant role in the $N^*(1535) \rightarrow N\rho$ partial decay width [15, 16]. We therefore retain only the S -wave term with a Lagrangian of the form

$$\mathcal{L}_{\rho NN^*} = ig_{N^*N\rho}\bar{u}_N\gamma_5\left(\gamma_\mu - \frac{q_\mu\not{q}}{q^2}\right)\vec{\tau}\cdot\vec{\rho}^\mu u_{N^*} + h.c., \quad (1)$$

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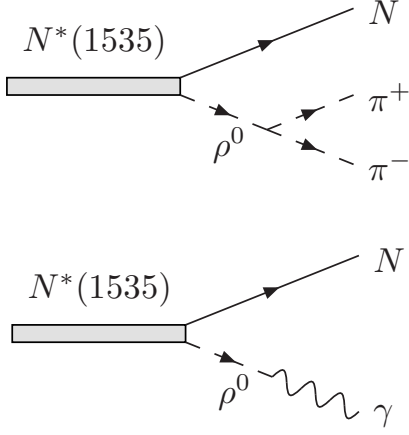


FIG. 1: Feynman diagrams considered for the $N^*(1535) \rightarrow N\rho^0 \rightarrow N\pi^+\pi^-$ and $N^*(1535) \rightarrow N\rho^0 \rightarrow N\gamma$ decays.

where u_N and u_{N^*} are the nucleon and $N^*(1535)$ spinors and q is the isobar four-momentum. The ρ -meson field $\vec{\rho}_\mu$ is also a vector in isospin space and $\vec{\tau}$ is the isospin operator in the baryon sector. It is seen that this form is a particular linear combination of vector and tensor couplings.

The finite size of the hadrons is taken into account through a form factor which is normalized to unity at $p_\rho^2 = m_\rho^2$. Since only the S -wave is involved, this is taken to be monopole type

$$F(p_\rho^2) = \frac{\Lambda^2}{\Lambda^2 + |p_\rho^2 - m_\rho^2|}, \quad (2)$$

with a cut-off parameter Λ . In the case of t -channel exchange, $p_\rho^2 < m_\rho^2$, it leads to the more familiar form

$$F(p_\rho^2) = \frac{\Lambda_t^2 - m_\rho^2}{\Lambda_t^2 - p_\rho^2}, \quad (3)$$

with $\Lambda_t^2 = \Lambda^2 + m_\rho^2$.

For the $\rho\pi\pi$ and $\rho\gamma$ couplings, we use the standard Lagrangians [17, 18, 19],

$$\mathcal{L}_{\rho\pi\pi} = g_{\rho\pi\pi}(\vec{\pi} \times \partial^\mu \vec{\pi}) \cdot \vec{\rho}_\mu, \quad (4)$$

$$\mathcal{L}_{\rho\gamma} = \frac{em_\rho^2}{f_\rho} \rho_\mu^0 A^\mu. \quad (5)$$

where $\vec{\pi}$ and A^μ are the pion and electromagnetic fields, respectively. The direct photon-vector coupling in Feynman diagram language is reflected in the factor em_ρ^2/f_ρ .

The value of the $\rho\pi\pi$ coupling constant $g_{\rho\pi\pi}$ can be deduced from the partial decay width

$$\Gamma_{\rho^0 \rightarrow \pi^+\pi^-} = \frac{g_{\rho\pi\pi}^2}{6\pi} \frac{(p_\pi^{\text{cm}})^3}{m_\rho^2}, \quad (6)$$

where p_π^{cm} is the momentum of one of the pions in the rest frame of the ρ -meson. The experimental data then yield $g_{\rho\pi\pi}^2/4\pi = 2.91$.

Many photoproduction reactions have been successfully related to ones involving the production or decay of vector mesons within the vector meson dominance model. As a consequence, there are several ways to evaluate the $\rho\gamma$ coupling constant but they differ little from those given in the original Sakurai compilation [20] and we take $f_\rho^2/4\pi = 2.7$.

The amplitude for the strong decay $N^*(1535) \rightarrow N\rho^0 \rightarrow N\pi^+\pi^-$ has the form

$$\mathcal{M}_{N^* \rightarrow N\rho^0 \rightarrow N\pi^+\pi^-} = ig_{\rho\pi\pi} g_{N^*N\rho} F(p_\rho^2) \times \bar{u}_N \gamma_5 \left(\gamma^\mu - \frac{\not{q} \gamma^\mu}{q^2} \right) u_{N^*} G_{\mu\nu}^\rho(p_\rho) (p_2^\nu - p_3^\nu), \quad (7)$$

Here $G_{\mu\nu}^\rho(p_\rho)$ is the ρ -meson propagator,

$$G_{\mu\nu}^\rho(p_\rho) = -i \frac{g^{\mu\nu} - p_\rho^\mu p_\rho^\nu / p_\rho^2}{p_\rho^2 - m_\rho^2 + im_\rho \Gamma_\rho}, \quad (8)$$

where Γ_ρ is the total ρ decay width.

The partial decay width is related to the spin-averaged amplitude through

$$d\Gamma_{N^* \rightarrow N\rho^0 \rightarrow N\pi^+\pi^-} = |\overline{\mathcal{M}_{N^* \rightarrow N\rho^0 \rightarrow N\pi^+\pi^-}}|^2 \times \frac{m_N}{(2\pi)^5} \frac{d^3 p_1 d^3 p_2 d^3 p_3}{4E_1 E_2 E_3} \delta^4(M_{N^*} - p_1 - p_2 - p_3), \quad (9)$$

where p_1 , p_2 , p_3 and E_1 , E_2 , E_3 are the momenta and energies of the nucleon, π^+ , and π^- , respectively.

The phase-space integration of Eq. (9) was evaluated numerically and the values of the cut-off parameter of Eq. (2) and the $N^*(1535)N\rho$ coupling constant adjusted to yield the experimental partial width of (3.0 ± 1.6) MeV which is obtained from the PDG values for the total decay width of 150 ± 25 MeV and the branching ratio of 0.02 ± 0.01 [16]. In Fig. 2 the value of $g_{N^*N\rho}^2/4\pi$ is shown as a function of Λ by the dashed curve. In view of the uncertainty in the partial width, one should consider an error corridor of $\pm 53\%$ around this curve.

Turning now to the radiative decay, the current best PDG estimates of the helicity- $\frac{1}{2}$ decay amplitudes for the charged and neutral $N^*(1535)$ are $A_{1/2}^{p\gamma} = 0.090 \pm 0.030 (\text{GeV})^{-1/2}$ and $A_{1/2}^{n\gamma} = -0.046 \pm 0.027 (\text{GeV})^{-1/2}$, respectively [16]. These lead to the corresponding isovector helicity- $\frac{1}{2}$ decay amplitude of the $N^*(1535)$ as

$$A_{1/2}^{I=1} = \frac{1}{2} (A_{1/2}^{p\gamma} - A_{1/2}^{n\gamma}) = (0.068 \pm 0.020) (\text{GeV})^{-1/2}, \quad (10)$$

in terms of which the $N^*(1535) \rightarrow N\gamma$ partial decay width for isovector photons becomes

$$\Gamma_{N^*N\gamma} = \frac{k^2}{\pi} \frac{m_N}{M_{N^*}} (A_{1/2}^{I=1})^2, \quad (11)$$

where k is the photon momentum in the N^* rest frame.

The radiative decay width can be estimated within the VMD model by applying the Feynman rules to Fig. 1b.

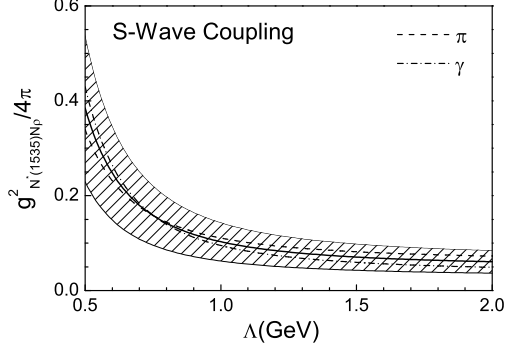


FIG. 2: Coupling constant $g_{N^*(1535)N\rho}^2/4\pi$ versus the cut-off parameter Λ for the pure S -wave coupling case. The dashed curve was obtained from the $N^* \rightarrow N\pi\pi$ decay whereas the dot-dashed one corresponds to the $N^* \rightarrow N\gamma$ decay. The solid curve represents the average of the two approaches with the shading showing the uncertainties arising from the errors in the experimental input.

The resulting matrix element is

$$\mathcal{M}_{N^* \rightarrow N\gamma} = -i \frac{em_\rho^2}{f_\rho} g_{N^*N\rho} F(p_\rho^2) G_{\mu\nu}^\rho(p_\rho) \varepsilon^\nu(k) \times \bar{u}_N \gamma_5 \left(\gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) u_{N^*}, \quad (12)$$

where $\varepsilon^\nu(k)$ is the polarization vector of photon. The resulting decay width is

$$\Gamma_{N^* \rightarrow N\gamma} = \frac{g_{N^*N\rho}^2}{4\pi} \frac{\alpha}{f_\rho^2/4\pi} \frac{3k(m_N + E_N)}{M_{N^*}(1 + \Gamma_\rho^2/m_\rho^2)(1 + m_\rho^2/\Lambda^2)^2}, \quad (13)$$

where α is the fine-structure constant and E_N the energy of the final nucleon. The numerical value of $A_{1/2}^{I=1}$ from Eq. (10) leads to the dot-dashed curve of Fig. 2, which shows $g_{N^*N\rho}^2/4\pi$ versus Λ as derived from the radiative decay. An uncertainty corridor must also be associated with this curve because of the large error in the radiative amplitude shown in Eq.(10).

The values of $g_{N^*N\rho}^2/4\pi$ extracted from the two decays are mutually compatible within the error bars for the whole range of Λ from 0.5 to 2.0 GeV. From these two independent measurements we deduce the average value and the corresponding uncertainty corridor, as shown by the solid curve and the shaded area in Fig. 2.

We can derive analogous constraints from these data on the other two commonly used forms for the $N^*(1535)N\rho$ coupling, i.e., pure vector or pure tensor which have, respectively, the corresponding effective Lagrangians

$$\mathcal{L}_{\rho NN^*}^V = ig_{N^*N\rho} \bar{u}_N \gamma_5 \gamma_\mu \vec{\tau} \cdot \vec{\rho}^\mu u_{N^*} + h.c., \quad (14)$$

$$\mathcal{L}_{\rho NN^*}^T = i \frac{g_{N^*N\rho}}{2m_N} \bar{u}_N \gamma_5 \sigma_{\mu\nu} \partial^\nu \vec{\tau} \cdot \vec{\rho}^\mu u_{N^*} + h.c.. \quad (15)$$

Since these two kinds of coupling involve both S -wave and D -wave, a dipole form factor is used for the $N^*N\rho$

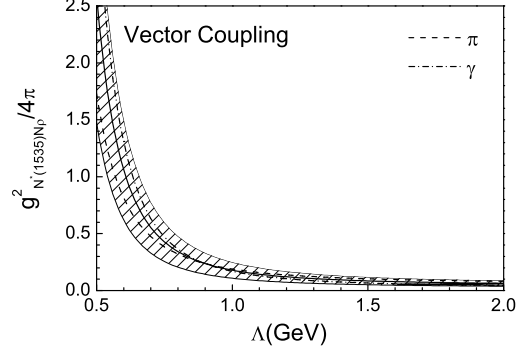


FIG. 3: As for Fig. 2 but for the pure vector coupling case.

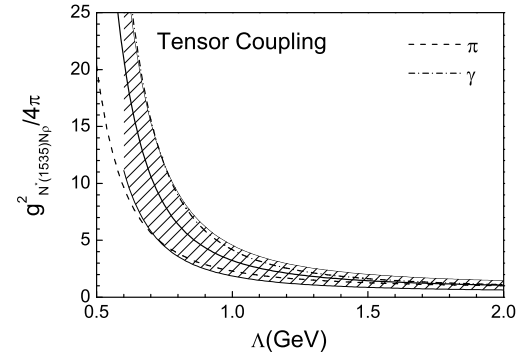


FIG. 4: As for Fig. 2 but for the pure tensor coupling case.

vertex:

$$F(p_\rho^2) = \left(\frac{\Lambda^2}{\Lambda^2 + |p_\rho^2 - m_\rho^2|} \right)^2. \quad (16)$$

The corresponding results are shown in Fig. 3 and Fig. 4, respectively. For the pure vector coupling case, the extracted values from the two decays are also agree within error bars for the whole range of Λ from 0.5 to 2.0 GeV, while for pure tensor coupling this is only true for $\Lambda > 0.6$ GeV.

III. DISCUSSION AND CONCLUSIONS

In this short note we have compared the values obtained for the $N^*(1535)N\rho$ coupling constant from experimental data on the radiative and two-pion decays of $N^*(1535)$ resonance. For this purpose we have used an effective Lagrangian approach combined with the vector meson dominance model that links photoproduction reactions to ones involving the ρ and other vector mesons. With a particular choice of the form of the $N^*(1535)N\rho$ vertex (S -wave coupling), we show in Fig. 2 that the two determinations are quite compatible for a wide range of

the cut-off parameter Λ , especially if account is taken of the error bands that arise from uncertainties in the input data. Typically one would expect $\Lambda_t = (\Lambda^2 + m_\rho^2)^{1/2}$ to be of the order of $1 \text{ GeV}/c^2$ [21], which falls well within the domain of compatibility.

It is seen from Figs. 3 and 4 that the pure vector and tensor forms of the coupling can also reproduce simultaneously the data within the rather large error bars, though marginally worse than the pure S -wave coupling of Fig. 2. Both vector and tensor forms are linear combinations of S -wave and D -wave couplings. However, since both give only a small D -wave contribution to $N^*(1535) \rightarrow N\rho$, the available data are not precise enough to discriminate between them. One can only put constraint on their couplings versus the cut-off parameter Λ , as shown by Figs. 2-4. If the data on both the two-pion and radiative decays were improved significantly, one might eventually hope to identify unambiguously the form of the N^* coupling from a comparison of the two rates.

In conclusion, the $N^*(1535)N\rho$ vertex can be constrained by the available experimental data from the ra-

diative and two-pion decays of $N^*(1535)$ resonance. The pure S -wave coupling gives a good simultaneous fit to the data, though the large error bars means that one cannot exclude either the pure vector or tensor forms. The values of the coupling constant are strong in the sense that they would predict a large ρ -exchange contribution to η production in nucleon-nucleon scattering [8, 9, 10, 11, 12] so that it would be very unwise to neglect it.

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